GENERALIZED COUETTE FLOW OF A NONLINEARLY VISCOPLASTIC FLUID

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The generalized Couette flow of a nonlinearly viscoplastic fluid is studied theoretically.

Let us consider the established flow of a nonlinearly viscoplastic fluid between two plates, the upper of which is moving with a constant velocity U in the direction of the Ox axis. A constant pressure gradient grad p=A acts in the gap. Its origin can be due to mechanical or other causes, such as the action on a ferromagnetic suspension of a magnetic field running along the channel axis. The direction of the velocity vector $\vec{U}$ can coincide with $\vec{A}$ or be opposite to it (Fig. 1).

The following three modes of flow are possible, depending on the rheophysical parameters of the fluid, the magnitude and direction of the pressure gradient, and the velocity U:

1) flow with a quasisolid zone (the core) within the stream;

2) flow with a core adjacent to the upper or lower plate;

3) flow without a core.

To describe the rheological behavior of the fluid we use the generalized model [1]

$$\tau^{\frac{1}{n}} = \tau_0^{\frac{1}{n}} + (\eta_p \dot{\gamma})^{\frac{1}{m}} \tag{0.1}$$

with the rheological parameters m, n, and η_p (all real numbers).

In the generalized coordinates $\tau^* = \tau/\tau_0$ and $\dot{\gamma}^* = \dot{\mu}\gamma/\tau_0$ the flow curve (0.1) is transformed to the form [2]

$$\tau^* = (1 + \ddot{\varkappa}\dot{\gamma}^{*\frac{1}{m}})^n. \tag{0.2}$$

Some particular forms of flow curves at the τ^* and $\dot{\gamma}^*$ axes are presented in Fig. 2. For clarity, the dimensional coefficient $\varkappa = \tau_0^{(n-m)/nm}$ is taken as equal to unity.

§ 1. The Core within the Stream

For the chosen statement of the problem and the conditions of attachment to both walls of the channel, the equation of motion is written in the form

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y}, \tag{1.1}$$

from which

$$\tau = A(y - y_0). \tag{1.2}$$

Here y_0 is the integration constant, which has the meaning of the coordinate of the plane in which the tangential shear stress equals zero.

If $U \neq 0$, then the core is located, generally speaking, asymmetrically relative to the midplane, and one must consider the two regions of shear flow (I: $\tau < 0$ and II: $\tau > 0$) and the zone of quasisolid flow III (Fig. 1) separately

$$\begin{array}{l} I:\tau<0,\ 0\leqslant y\leqslant y_1, \\ II:\tau>0,\ y_2\leqslant y\leqslant h, \end{array} \quad III:\begin{cases}\tau<0,\ y_1\leqslant y<y_0,\\ \tau=0,\ y=y_0,\\ \tau>0,\ y_0<y\leqslant y_2.\end{cases} \tag{1.3}$$

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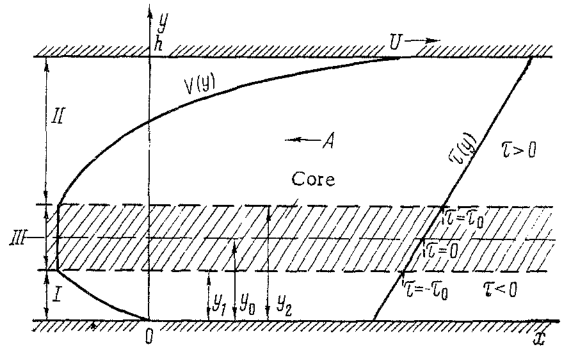

Fig. 1

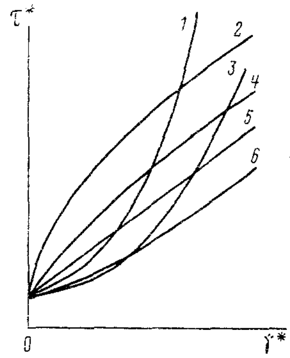

Fig. 2

Fig. 1. On the statement of the problem.

Fig. 2. Flow curves: 1) m=1; n=2; 2) m=n=2; 3) m=$^1/_2$; n=1; 4) m=2; n=1; 5) m=n=1; 6) m=n=$^1/_2$.

Allowing for the signs of τ for regions I and II, from (0.1) we obtain

$$I:\tau<0,\ (-\tau)^{\frac{1}{n}}=\tau_0^{\frac{1}{n}}+\left(-\eta_p\frac{dV_1}{dy}\right)^{\frac{1}{m}},$$

$$II:\tau>0,\ \tau^{\frac{1}{n}}=\tau_0^{\frac{1}{n}}+\left(\eta_p\frac{dV_2}{dy}\right)^{\frac{1}{m}}.$$

Let us formulate the problems in the dimensionless form

$$\alpha\frac{dW_1}{d\xi}=-[(\xi_0-\xi)^{\frac{1}{n}}-\beta_0^{\frac{1}{n}}]^m,\quad \alpha\frac{dW_2}{d\xi}=[(\xi-\xi_0)^{\frac{1}{n}}-\beta_0^{\frac{1}{n}}]^m \tag{1.4}$$

with the boundary conditions

$$W_1(0)=0,\ W_2(1)=1. \tag{1.5}$$

After simple transformations we obtain

$$\begin{gathered}\alpha W_1(\xi)=\sum_{k=0}^{\infty}(-1)^k C_m^k\frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[(\xi_0-\xi)^{\varepsilon_k}-\xi_0^{\varepsilon_k}],\ 0\leqslant\xi\leqslant\xi_1,\\ \alpha W_2(\xi)=\alpha+\sum_{k=0}^{\infty}(-1)^k C_m^k\frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[(\xi-\xi_0)^{\varepsilon_k}-(1-\xi_0)^{\varepsilon_k}],\ \xi_2\leqslant\xi\leqslant1,\end{gathered} \tag{1.6}$$

where

$$\varepsilon_k=\frac{m+n-k}{n},\ C_m^k=\frac{m!}{k!\,(m-k)!}. \tag{1.7}$$

When m and n are integers, in particular, the series in (1.6) are finite sums and one can obtain expressions for the velocity profiles in a clearer form (Table 1).

From the condition $\tau=\tau_0$ at $y=y_1$ and $y=y_2$ we obtain

$$\xi_0-\xi_1=\beta_0,\ \xi_2-\xi_0=\beta_0,\ \xi_2-\xi_1=2\beta_0. \tag{1.8}$$

It follows from (1.8) that in a mode of flow with a core within the stream the parameter ξ_0 also determines the middle of the core. And even if the ordinate ξ_0 does not coincide with the middle of the region of flow (an asymmetrical location of the core), the plane where the shear stress is equal to zero is still located in the middle of the core. We note also that in the mode of flow under consideration, with fixed conditions of flow the width of the core is constant and equal to $2\beta_0$ regardless of the position of the core within the channel.

To determine the value of ξ_0 we use the condition $W_1(\xi_1)=W_2(\xi_2)$, from which, on the strength of (1.8), we have

$$\alpha=\sum_{k=0}^{\infty}(-1)^k C_m^k\frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[(1-\xi_0)^{\varepsilon_k}-\xi_0^{\varepsilon_k}]. \tag{1.9}$$

TABLE 1. Expressions for the Velocity Distribution

m; / n;	$\alpha W_1(\xi)$	$\alpha W_2(\xi)$
1;/1;	$\frac{1}{2}\xi^2 + (\beta_0 - \xi_0)\xi$	$\frac{1}{2}\xi^2 - (\xi_0 + \beta_0)\xi + \left(\xi_0 + \beta_0 + \alpha - \frac{1}{2}\right)$
1;/2;	$\frac{2}{3}(\sqrt{\xi_0 - \xi} - \sqrt{\xi_0})(2\xi_0 - \xi + \sqrt{\xi_0(\xi_0 - \xi)} + \beta_0\xi)$	$\frac{2}{3}\sqrt{(\xi - \xi_0)^3} - \sqrt{(1 - \xi_0)^3} - \beta_0\xi + \alpha + \beta_0$
2;/1;	$-\frac{1}{3}\xi_0^3 + (\xi_0 - \beta_0)\xi^2 - (\xi_0 - \beta_0)^2\xi$	$\frac{1}{3}\xi_0^3 - (\xi_0 + \beta_0)\xi^2 + (\xi_0 + \beta_0)^2\xi + \left[-(\xi_0 + \beta_0)^2 + (\xi_0 + \beta_0) + \alpha - \frac{1}{3}\right]$
2;/2;	$\frac{1}{3}\xi^2 - (\xi_0 + \beta_0)\xi - \frac{4}{3}\sqrt{\beta_0} \times (\sqrt{\xi_0 - \xi} - \sqrt{\xi_0})(2\xi_0 - \xi + \sqrt{\xi_0(\xi_0 - \xi)})$,	$\frac{1}{2}\xi^2 + (\beta_0 - \xi_0)\xi + \left(\alpha - \beta_0 - \frac{1}{2}\right) - \frac{4}{3}\sqrt{\beta_0}(\sqrt{\xi - \xi_0} - \sqrt{1 - \xi_0})(\xi - 2\xi_0 + 1 + \sqrt{(\xi - \xi_0)(1 - \xi_0)})$

Now let us determine the region of variation of the quantities α and β_0 within which flow with a core within the stream is realized. The parameter β_0 determines the width of the core while the value of α characterizes its position. For a fixed value of β_0 upon an increase in α (i.e., in the velocity of the upper plate) the core drops down, retaining its relative width. The value at which the core is adjacent to the lower plate is determined from (1.9). Taking $\xi_1 = 0$, $\xi_0 = \beta_0$, and $\xi_2 = 2\beta_0$, we have

$$\alpha_{\text{low}} = \sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [(1 - \beta_0)^{\varepsilon_k} - \beta_0^{\varepsilon_k}]. \tag{1.10}$$

For negative α the core rises up with an increase in $|\alpha|$. The value of α at which the core is adjacent to the upper plate is determined from the condition $\xi_2 = 1$, $\xi_0 = 1 - \beta_0$, and $1 - 2\beta_0 = \xi_1$, from which we have

$$\alpha_{\text{up}} = -\sum_{k=1}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [(1 - \beta_0)^{\varepsilon_k} - \beta_0^{\varepsilon_k}]. \tag{1.11}$$

For a stationary upper plate ($\alpha = 0$) we have $\xi_0 = 0.5$; i.e., the core is located symmetrically relative to the midplane. For values of α which are equal in absolute value but opposite in sign the displacements of the core upward and downward are symmetrical relative to the midplane $\xi = 0.5$.

The case of $\alpha = 0$ has the meaning of confined quasi-Poiseuille flow in a channel with stationary walls. In the case when the core lies within the region of flow the equation for the determination of ξ_0 takes the form

$$\sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [(1 - \xi_0)^{\varepsilon_k} - \xi_0^{\varepsilon_k}] = 0.$$

For this case with $n > 1$ and any m there is one solution $\xi_0 = 0.5$ and, depending on the value of β_0, two other solutions of the form

$$\xi_{02-3} = 0.5 \pm f_{mn}(\beta_0),$$

where $f_{mn}(\beta_0)$ is some nonnegative function of β_0 whose form depends on the values of m and n. In the case of $n \leq 1$ (the presence of only one solution $\xi_0 = 0.5$) the core is symmetrically located in the region of flow and has a width $h_1 = 2\beta_0$. In the presence of the solutions ξ_{02-3} the core is symmetrically located but now it has a width $h_2 = 2(\beta_0 + f_{mn}(\beta_0))$. (We note that $f_{mn}(\beta_0) \leq \beta_0$. And furthermore, when $\beta_0 \geq 0.5$ the core fills the entire region of flow.)

Let us present examples for the solutions ξ_{02-3} with $\alpha = 0$:

1) For

$$m = 1,\ n = 2 \quad f_{12} = \sqrt{\frac{1}{4} - d},\ d = (2a^2 + a) - 2a\sqrt{(a + 1)a},\ a = \beta_0 - \frac{4}{9},\ \left(\beta_0 \geq \frac{4}{9}\right);$$

2) for

$$m = n = 2 \quad f_{22} = \sqrt{\frac{1}{4} - g}, \; g = \frac{c(b+2) + 2c\sqrt{b+1}}{b^2}, \; b = \frac{16}{9}\beta_0, \; c = \left(\beta_0 + \frac{1}{2}\right)^2.$$

§2. The Core Adjacent to One of the Plates

If the core is adjacent to the upper wall, $\alpha < 0$, the velocity profile is determined from the equation

$$W_{\text{up}}(\xi) = \frac{1}{\alpha}\sum_{k=0}^{\infty}(-1)^k C_m^k \frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[(\xi_0 - \xi)^{\varepsilon_k} - \xi_0^{\varepsilon_k}]. \tag{2.1}$$

The constant ξ_0 is determined from the condition $W_{\text{up}}(\xi_1) = 1$. Since $\xi_0 - \xi_1 = \beta_0$, for ξ_0 we have

$$\alpha = \sum_{k=0}^{\infty}(-1)^k C_m^k \frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[\beta_0^{\varepsilon_k} - \xi_0^{\varepsilon_k}]. \tag{2.2}$$

If the core is adjacent to the lower plate ($\alpha > 0$), then the velocity profile is determined by the expression

$$W_{\text{low}}(\xi) = 1 + \frac{1}{\alpha}\sum_{k=0}^{\infty}(-1)^k C_m^k \frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[(\xi - \xi_0)^{\varepsilon_k} - (1 - \xi_0)^{\varepsilon_k}]. \tag{2.3}$$

The constant ξ_0 is determined from the condition $W_{\text{low}}(\xi_2) = 0$. Since $\xi_2 - \xi_0 = \beta_0$, we have

$$\alpha = \sum_{k=0}^{\infty}(-1)^k C_m^k \frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[(1 - \xi_0)^{\varepsilon_k} - \beta_0^{\varepsilon_k}]. \tag{2.4}$$

We note that ξ_0 is no longer the middle of the core actually remaining in the region of flow but of a core of width $2\beta_0$ extending through the plate, as it were. Therefore, one allows $\xi_0 > 1$ for (2.2) and $\xi_0 < 0$ for (2.4).

The case when $\xi_0 = 0$ and $\xi_0 = 1$, i.e., when ξ_0 coincides with the surfaces of the lower and upper plates, respectively, and the shear stress τ conserves its sign everywhere in the channel, is of considerable interest.

If $\xi_0 = 1$ and $\alpha < 0$ then the corresponding α is

$$\alpha|_{\xi_0=1} = \sum_{k=0}^{\infty}(-1)^k C_m^k \frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[\beta_0^{\varepsilon_k} - 1]. \tag{2.5}$$

Similarly,

$$\alpha|_{\xi_0=0} = -\alpha|_{\xi_0=1}. \tag{2.6}$$

§3. Flow without a Core

This mode occurs when the core passes beyond the limits of the channel, as it were. Two cases should be distinguished: $\alpha > 0$ and $\alpha < 0$, i.e., when the core passes beyond the lower or the upper plate.

If $\alpha < 0$, then the velocity profile is determined from (2.1). To find ξ_0 we use the boundary condition $W_{\text{up}}(1) = 1$, from which

$$\alpha_{\text{up}} = \sum_{k=0}^{\infty}(-1)^k C_m^k \frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[(\xi_0 - 1)^{\varepsilon_k} - \xi_0^{\varepsilon_k}]. \tag{3.1}$$

If $\alpha > 0$, then from the condition $W_{\text{low}}(0) = 0$ we get

$$\alpha_{\text{low}} = -\alpha_{\text{up}}. \tag{3.2}$$

The limiting values of ξ_0 are $\xi_0 = 1 + \beta_0$ ($\xi_1 = 1$) and $\xi_0 = -\beta_0$ ($\xi_2 = 0$). Accordingly,

$$\alpha|_{\xi_0=1+\beta_0} = \sum_{k=0}^{\infty}(-1)^k C_m^k \frac{1}{\varepsilon_k}\beta_0^{\frac{k}{n}}[\beta_0^{\varepsilon_k} - (1 + \beta_0)^{\varepsilon_k}] = -\alpha|_{\xi_0=-\beta_0}. \tag{3.3}$$

With a further increase in $|\alpha|$ ($\xi_0 > 1 + \beta_0$, $\xi_0 < -\beta_0$) the "core" gradually departs beyond the plate to infinity.

In the limiting case when $\xi_0 = \pm\infty$ a linear velocity profile is established in the gap.

TABLE 2. Equations of the Characteristic Curves

$m;/n;$	Γ_1	Γ_3	S_1
0,5;/0,5;	$\alpha = \frac{1-\beta_0}{2}\sqrt{1-2\beta_0} + \frac{\beta_0^2}{2}\ln\left\|\frac{\beta_0}{1-\beta_0+\sqrt{1-2\beta_0}}\right\|$	$\alpha = -\frac{\beta_0^2}{2} \times \ln\left\|\frac{1+\beta_0+\sqrt{1+2\beta_0}}{\beta_0}\right\| + \frac{1+\beta_0}{2}\sqrt{1+2\beta_0}$	$\alpha = -\frac{\beta_0^2}{2} \times \ln\left\|\frac{1+\sqrt{1+\beta_0^2}}{\beta_0}\right\| + \frac{1}{2}\sqrt{1-\beta_0^2}$
0,5;/1;	$\alpha = \frac{2}{3}\sqrt{(1-2\beta_0)^3}$	$\alpha = \frac{2}{3}$	$\alpha = \frac{2}{3}\sqrt{(1-\beta_0)^3}$
1;/0,5;	$\alpha = \frac{1}{3}[(1-\beta_0)^3 - \beta_0^3] - \beta_0^2[1-2\beta_0]$	$\alpha = \frac{1}{3} + \beta_0$	$\alpha = \frac{2}{3}\beta_0^3 + \beta_0^2 + \frac{1}{3}$
1;/1;	$\alpha = \frac{1}{2}[1-2\beta_0)^2$	$\alpha = \frac{1}{2}$	$\alpha = \frac{1}{2}(1-\beta_0)^2$
2;/1;	$\alpha = -\frac{7}{3}\beta_0^3 + 4\beta_0^2 - 2\beta_0 + \frac{1}{3}$	$\alpha = \frac{1}{3}$	$\alpha = -\frac{1}{3}\beta_0^3 + \beta_0^2 - \beta_0 + \frac{1}{3}$
1;/2;	$\alpha = \frac{2}{3}[(1-\beta_0)^{\frac{3}{2}} - \beta_0^{\frac{3}{2}}] - \sqrt{\beta_0}(1-2\beta_0)$	$\alpha = \frac{2}{3}[(1+\beta_0)^{\frac{3}{2}} - \beta_0^{\frac{3}{2}}] - \sqrt{\beta_0}$	$\alpha = \frac{2}{3}[1-\beta_0^{\frac{3}{2}}] - \beta_0^{\frac{1}{2}}(1-\beta_0)$
2;/2;	$\alpha = \frac{1}{2}(1-4\beta_0^2) - \frac{4}{3}\sqrt{\beta_0}[(1-\beta_0)^{\frac{3}{2}} - \beta_0^{\frac{3}{2}}]$	$\alpha = (2\beta_0+1) - \frac{4}{3}\sqrt{\beta_0}[(1+\beta_0)^{\frac{3}{2}} - \beta_0^{\frac{3}{2}}]$	$\alpha = -\frac{1}{6}\beta_0^2 + \beta_0 - \frac{4}{3}\sqrt{\beta_0} + \frac{1}{2}$

§ 4. Regions of the Mode of Flow

We designate as D_1 the region of flow with the core within the stream; D_2 is that with the core at the lower plate; D_3 with the core adjacent to the upper plate; D_4 with the core "gone" beyond the lower plate; D_5 with the core "gone" beyond the upper plate.

The curves Γ_1 and Γ_2 separating the region of flow with the core within the stream are determined from the conditions (1.10) and (1.11)

$$\Gamma_1 : \alpha = \sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [(1-\beta_0)^{\varepsilon_k} - \beta_0^{\varepsilon_k}],$$

$$\Gamma_2 : \alpha = -\sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [(1-\beta_0)^{\varepsilon_k} - \beta_0^{\varepsilon_k}]$$

(here β_0 is considered not as a fixed but as a variable quantity). The curves Γ_3 and Γ_4 separating the regions of flow with a core adjacent to one of the plates and the regions of flow without a core are determined from (3.3) and (3.4)*:

$$\Gamma_3 : \alpha = -\sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [\beta_0^{\varepsilon_k} - (1+\beta_0)^{\varepsilon_k}],$$

$$\Gamma_4 : \alpha = \sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [\beta_0^{\varepsilon_k} - (1+\beta_0)^{\varepsilon_k}].$$

The curves Γ_1 and Γ_2 as well as Γ_3 and Γ_4 are symmetrical to each other relative to the straight line $\alpha = 0$ in the plane $0\alpha\beta_0$.

*Sentence as in Russian original, no Eq. (3.4) occurs in original article – Publisher.

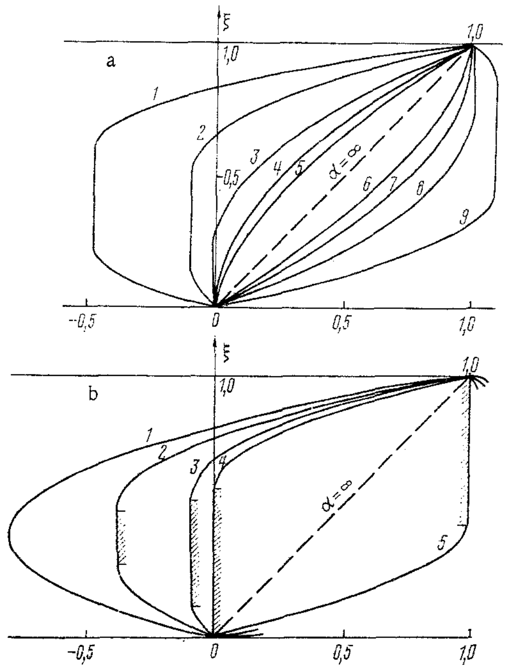

Fig. 3. Profiles of flow velocity: a) $\beta_0 = 0.2$: 1) $\alpha = 0.05$; 2) $\alpha = 0.1$; 3) $\alpha = 0.3$; 4) $\alpha = 0.5$; 5) $\alpha = 0.7$; 6) $\alpha = -0.7$; 7) $\alpha = -0.5$; 8) $\alpha = -0.3$; 9) $\alpha = -0.1$; b) $\alpha = 0.1$: 1) $\beta_0 = 0$; 2) $\beta_0 = 0.1$; 3) $\beta_0 = 0.2$; 4) $\beta_0 = 0.4$; 5) $\alpha = -0.1$, $\beta_0 = 0.4$. $m = n = 1$.

Starting with the time when $\xi_0 = 0$ (or $\xi_0 = 1$), the shear stress τ conserves its sign over the entire width of the channel, namely, $\tau > 0$ ($\tau < 0$). In this connection, in addition to these curves one can introduce the characteristic curves S_1 and S_2 which determine the region of sign-constancy of τ in the plane $0\alpha\beta_0$. By virtue of (2.5) and (2.6) we have

$$S_1 : \alpha = \sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [1 - \beta_0^{\varepsilon_k}],$$

$$S_2 : \alpha = -\sum_{k=0}^{\infty} (-1)^k C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}} [1 - \beta_0^{\varepsilon_k}].$$

The form of the equations of the curves Γ_1, Γ_3, and S_1 for some concrete values of m and n are presented in Table 2. These curves characterize the critical relationships between the velocity U and pressure gradient A and the rheological characteristics η_p and τ_0 (for fixed m and n). We note that the curves Γ_3 and Γ_4 diverge from the $0\beta_0$ axis for $n < 1$, are parallel to the axis for $n = 1$ (a kind of "boundary situation"), while they converge asymptotically to the $0\beta_0$ axis for $n > 1$.

The velocity profiles are presented in Fig. 3.

For the flow of a specific medium in a certain channel (τ_0, η_p, m, and n are known constants and the channel width h is given) the condition $\beta_0 = \text{const}$ is equivalent to the condition that $\text{grad } p = A$ is a fixed quantity. And then the variables α are equivalent to the variables U. The distribution of the velocity profiles as a function of the velocity U of the upper plate for a fixed pressure gradient is shown in Fig. 3a. Similarly, the distribution of the velocity profiles as a function of A for a fixed velocity U is given in Fig. 3b. The width of the core is proportional to A^{-1}.

The criteria α and β_0 can also serve for the estimation of the mode of flow in a channel when the pressure gradient grad p is variable. For example, for any m and n, flow in a mode with a core within the stream is possible if

$$\beta_0^* = \frac{\tau_0}{h\,|\text{grad} p|} < 0.5. \tag{4.1}$$

If $n \geq 1$, then when

$$\alpha^* = \frac{\eta_p U}{h\,(|\text{grad } p| h)^{\frac{m}{n}}} > 0.5, \tag{4.2}$$

only a mode of flow without a core is possible. These results emerge directly from the properties of the curves Γ constructed in the axes α and β_0 for the case of grad p = const.

NOTATION

Dimensional quantities: U, velocity of upper plate; A, pressure gradient; τ_0, limiting shear stress; η_p, analog of plastic viscosity; m, n, nonlinearity parameters of flow curve; h, channel width; y_1, y_2, boundaries of core; V(y), flow velocity; $\dot{\gamma}$, shear velocity. Dimensionless quantities: W = V/U, flow velocity; $\xi = y/h$, vertical coordinate; ξ_1, ξ_2, boundaries of core; ξ_0, coordinate of the plane in which the shear stress equals zero; $\beta = \tau/Ah$, reduced shear stress; $\alpha = \eta_p U/(Ah)^{\frac{m}{n}}$ and $\beta_0 = \tau_0/Ah$, parameters.

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STABILITY OF OPERATION OF AN APPARATUS CONTAINING A GRANULAR BED FLUIDIZED BY A GAS STREAM

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The results of numerical experiments on the investigation of the stability of the fluidization process relative to finite perturbations and its behavior upon crossing the boundary of stability are presented.

In [1] the problem of the stability of the fluidization process was formulated in a framework within which the fluidized bed was considered as a single structureless element with certain operating characteristics, and the boundary of the region of stability in the space of the parameters of the process was studied in a linear approximation.

The present report is a continuation of [1]. The stability of the fluidization process relative to finite perturbations is demonstrated by a numerical experiment and its behavior upon crossing the boundary of the region of stability is studied.

In [1] a model of a fluidized bed was proposed which is described by the following equations:

$$\frac{1}{2} m\ddot{H} + mg + k_1(q + q_v) + k_2 q = p^* - p^0, \tag{1}$$

$$q_v = cV\left(\frac{1}{2} m\dddot{H} + k_2\dot{q}\right), \tag{2}$$

$$q = q_0 + \rho S\dot{H} + \sigma\frac{H - H_0}{H}. \tag{3}$$

From the system (1)-(3) we get the equation

$$\dddot{H} + a_1\ddot{H} + (a_2 + a_3H^{-2})\dot{H} + a_4H^{-1} + a_5 = 0, \tag{4}$$

where

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